Readiness Skills for Multidigit Number Processing

Before a child can read a book, he or she must acquire certain readiness skills, such as a sense of oral language and an understanding of how to orient a book. Just as there are readiness skills for the task of reading books, there are similar readiness skills for the arduous processes of multidigit computation.

Group Theory, Cardinality, and Number Sense

Before they can accurately manipulate groups of items, students must become aware that these groups exist in everyday life. Students need exposure to the process of organizing and compartmentalizing similar items in groups of discernible sizes. Students must then develop an understanding of the relationships between the quantities of items in these groups and the total number of items. Students can then move on to expressing these relationships as number facts.

Students must be led to append numbers to quantities they routinely encounter—for example, the number of ears on a rabbit, wheels on a tricycle, paws on a cat, toes on a foot, and so on. Once a student knows these quantities, he or she can begin to see things in terms of finite groups instead of loosely defined quantities, such as "a lot". These finite groups establish a critical knowledge bank from which the student can make comparisons to similar groups of items.

Additionally, the concept of multiplication demands the acknowledgement of sets of elements, all of which contain the same number of elements duplicated a number of times (×). Some students need direct instruction to acknowledge, label, and quantify grouped objects that are already present within their experiential world.

The examples that follow provide lasting, durable images that you may use later to give semantically meaningful examples of multiplication, division, and the encoding of fractions. Instruction should include the pairing of a common object with its enumerated attributes (parts).

EXAMPLE

bikes: 2 tires	**triangles: 3 sides**	**cows: 4 hooves**	**hand: 5 fingers**
people: 2 eyes	**tricycles: 3 wheels**	**cars: 4 tires**	**foot: 5 toes**
rabbits: 2 ears	**days: 3 meals**	**square: 4 sides**	**starfish: 5 legs**

In mathematics, the cardinality of a set is a measure of the number of elements of the set. Rather than trying to establish cardinality by counting objects from part to whole in a linear fashion, consider the benefit realized by counting the elements of a recognized set from whole to part. For instance, envision counting a handful of cereal piece by piece to determine the quantity—"one, two, three, four, five," and so on—versus looking at a starfish and counting its five legs. Which process does a better job of establishing a cardinal understanding of the number 5? Whole-to-part processing models provide the ability to integrate parts within the context of a whole number to establish cardinality.

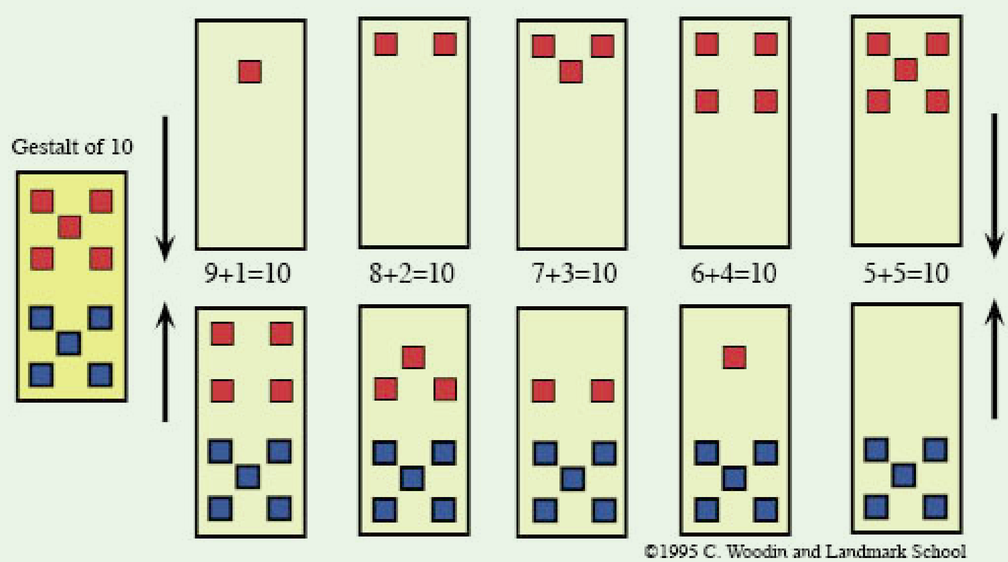
These whole-to-part models provide necessary durable semantic examples of nouns paired with their attributes, but it is difficult to compare the relative quantities of attributes between different facts. It is hard to establish a relative comparison between starfish legs and tricycle wheels. For this reason, although semantic whole-to-part relationships can be used to introduce cardinality, other standardized models must be implemented to promote a relational understanding of fact information.

Many students develop a dependence on sequential or part-to-whole counting strategies to perform addition. Students are typically taught to use a counting strategy to add single-digit numbers. Many children implicitly pick up patterns and consequently learn addition facts through familiarity. Some do not, even when provided with endless repetition. These students benefit from differentiated instruction that allows them to quantify and compare relative quantities in a whole-to-part manner.

The part-to-whole counting method is similar to sounding out a word phonetically. It is an effective, and perhaps initially necessary, strategy for students at lower levels. Yet, consider the efficiency of recognizing syllables or entire words on sight, rather than synthesizing words from individual sounds. It is inefficient merely to become a proficient counter. Just as learning-disabled students strive to recognize patterns among familiar words on sight, they must be taught to recognize and envision patterns among numbers.

Consistent graphic organizers that relate quantities to both 5 and 10 provide the structure necessary to establish one-to-one correspondences between numbers and discernible quantities and help students develop a relational understanding of the numbers 1 through 10. These visual models also provide a way to extend this knowledge to the base-ten system and multidigit computations. Number sense is developed through the process of assigning values to groups of objects and then making comparisons between these groups. Consider the following patterns that relate to the gestalts of 5 and 10.

ADDENDS TO TEN IN ICON FORMAT



Visual Clustering

Children's exposure to number-related dot patterns (patterns on dice, playing cards, and dominoes) has diminished as pastimes that feature cards and dice have been replaced by electronic games and television. Students should be provided with opportunities to develop their ability to "see" numbers as recognizable clusters instead of collections of discrete elements to be counted.

The nonverbal visual channel may prove to be a relatively strong modality for some students. Displaying Arabic numerals in conjunction with familiar dot patterns helps strengthen the relationship between an Arabic numeral (2) and a concrete quantity (••). When visual representations (:: = : + :) are paired with facts expressed in Arabic numerals (4 = 2 + 2), the accompanying clusters of corresponding dots help to reinforce the facts in a visual yet nonverbal manner. This method of presentation is a boon to students who have difficulty processing auditory information. The icons allow visual processing to occur. Then, after students have processed the number relationship, they may label it and name it as a number fact.

EXAMPLE

The side of a dice showing six dots clearly depicts two subordinate groups of three (:::) within the gestalt of the quantity six (6 = 3 × 2). This number fact exemplifies the visual symmetry inherent in the even multiplication fact families, in which the same numbers of dots occur in each row. These iconic relationships may be further developed using the activities presented later in this book.

It is very important to play games that use objects that display familiar (canonical) patterns: dice, playing cards, and dominoes. Time spent in the classroom or during recess teaching students games involving these items is compounded when students can play these games at home. Two effective games are cribbage and bones. Cribbage is a sufficiently popular game to make an explanation of the rules unnecessary here. Bones, on the other hand, is a wonderful game that requires six dice, pencil and paper, and two to six players.

GAME

Bones

Bones is a dice game that allows students to develop visual memory, practice place value, and internalize the regrouping process. When playing the game in math class, do not permit students to write down scores that need to be added. Instead, have students retain and add the numbers mentally. Thinking in terms of place value will help students substantially, encouraging them to use their place value knowledge. This is not an original game—it has existed for years under several names like Zilch or Farkle! *This game can be played with two or more players.*

Materials:

- six dice (per group of players)
- pencil and paper to record totaled scores

Goal

The goal of the game is to be the first player to earn more than 5,000 points.

How to Earn Points:

When a player rolls the dice, different numbers are worth different amounts of points.

- A 1 is worth 100 points.
- A 5 is worth 50 points.
- A 2, 3, 4, or 6 is not worth any points unless the number comes up on three dice in the same turn (three of a kind).
- When a player rolls three of a kind, he or she multiplies the number rolled by 100. For example, three 2s would be worth 200 points (100 × 2 = 200). An additional (fourth) 2 would not be worth additional points.
- Rolling a straight with all six dice (1, 2, 3, 4, 5, 6) is worth 1,000 points.
- Rolling six dice in one toss that are all worth points (a combination of 5s, 1s, and/or 3 of a kind) is worth 1,000 points.

Overview:

During his or her turn, a player rolls the dice a number of times to accumulate the most points possible. As a player rolls numbers that earn points, he or she puts those particular dice aside, taking them out of play. These points are not secure until the player chooses to stop his or her turn and register the points with the scorekeeper. The player may choose to continue rolling remaining dice as long as each roll contains a scoring die or dice (1, 5, three of a kind). If a player chooses to roll and gets no points in the roll, he or she has rolled "bones." Rolling bones ends the turn, and all the points the player put aside during that turn are lost. A player may choose to stop rolling at any point to avoid bones and earn the points from dice that were put aside. At the end of a turn, the player must tell the scorekeeper how many points he or she rolled (so the scorekeeper isn't the only mathematician).

When a player surpasses 5,000 points on a complete turn, each other player has one complete turn to beat that score. If no player can beat that score, the player who surpassed 5,000 points wins and the game ends.

Example Play:

- To determine the starting player, each player rolls a die. The player who rolls the highest number goes first. Play continues clockwise.
- Player One rolls all six dice. He or she rolls a 1, 2, 3, 5, 5, and 6. The 1 and the two 5s are worth points if the player chooses to put them aside. The player chooses to put the 1 aside to count as 100 points. Note that the player does not need to put aside *all* scoring dice. In order to continue, however, at least one die must be put aside. Here, although the 5s could count as 50 points each, the player chooses to keep the dice in play in an attempt to earn more points.
- Player One rolls the five remaining dice. If at least one of these dice scores, he or she may continue to roll. Player One rolls a 3, 4, 4, 4, and 6 and chooses to put aside the three 4s as 400 points. At this time, Player

One has 500 points and decides not to roll the two remaining dice (to avoid rolling bones, which would eliminate all the points scored on the turn).

- Player One says the point total out loud ("I have 500 points"), and the scorekeeper records it. All players should be performing the calculations mentally to check Player One's addition. If a player miscalculates, the classmates should politely help!
- Play continues with each other player rolling and putting aside dice that score points. Each turn stops when a player chooses to register points or when he or she rolls bones.

Note: Once a die is put aside, its points are set. For example, if in three consecutive rolls a player puts aside a 5 each time (for a total of 150 points), he or she cannot later consider those three 5s three of a kind for 500 points.

- As rounds of play continue, players must add new points earned to the recorded points *without* the aid of pencil and paper. A player may look at the recorded score on the scorekeeper's sheet, but must do the addition computation mentally. The player says the new total out loud and the scorekeeper writes it down. Again, all students should be doing the mental calculations in order to check the math of other players.

Encourage students to think of the numbers as hundreds when the values become larger. For example, "900 + 300 = 1200" becomes "nine hundreds plus three hundreds equals twelve hundreds." Also, students may find it helpful to use the dice themselves to assist with adding. For example, in 850 + 450, have the student place the dice with the 5 (50 points) on top of the 850 on the scoring sheet and say, "900." The student will now have nine hundreds plus four hundreds, to equal thirteen hundreds.

Special Situations

- If three 1s are rolled, a player may choose to count them as 100 points three times, equaling 300 points, or as three of a kind, equaling 100 points (1 × 100 = 100). Counting three 1s as 100 points might be a useful strategy if the player doesn't want to pass the 5,000 threshold because he or she wants another full turn. (Another full turn might allow the player to earn far more than 5,000 points, making it very unlikely that another player could catch up.)
- If a player rolls a 1,000-point combination (a straight of six dice or all six dice earning points), he or she can put the points aside and continue rolling. That is, the player can end his or her turn and register the 1,000 points, or continue rolling the dice to earn more points, with the 1,000 points at risk should the player roll bones.

Ending the Game

Once a player surpasses 5,000 points, each other player has one turn to try to beat that total score. The highest total score wins!

Place Value

Students have limited success with mathematics involving large numbers until they gain an accurate understanding of place value. Understanding of place value in the base-ten system can be developed using concrete models. Base-ten blocks and similar commercial materials provide a means to demonstrate this process visually. Having students add 10 and then 11 to a number without paper is a good way to provide insight and instill familiarity with the base-ten system. Students quickly discover that it is easier to add 11 to a number by increasing the digit in the ones place by 1 and then increasing the digit in the tens place by 1 than to count upward 11 times from the number. Similarly, it is very easy to add 9 to a given number by first increasing the digit in the tens place by 1 and then counting back by 1 from the ones place.

Estimation skills should be tied directly to multidigit computations to instill a better understanding of large numbers. For example, before asking a student to perform a multidigit computation, provide a rationale for performing the task and ask for an approximate answer. Then examine the actual answer for acceptability in terms of the situation being modeled. Compare the answer to the initial estimate. If the student produces an estimate that is vastly different from the correct answer, provide a strategy to produce a better estimate. For instance, if a student is asked to estimate the difference between 600 and 401, and he or she responds with an estimate of 9, show the student that the difference between the digits in the hundreds place is most important to an accurate estimation.

To solidify the student's understanding, illustrate the point with base-ten blocks. Create each number (600 and 401) using the blocks, and place the blocks so that their place values align vertically. Have the student imagine the blocks to be quantities of chocolate. Ask the student to identify which of the types of blocks represents the largest amount of chocolate. It should be abundantly clear that the two hundreds (or flats) would yield the largest quantity; therefore, 200 would be a better estimate.

The game Big Number is an entertaining way for students to practice reading and writing multidigit numbers while developing place value, estimation, and comparison skills. Big number is a very easy game to prepare for and play. Students must use their knowledge of place value to construct the largest number possible from a series of digits. Base-ten Ball is another interactive way to practice place value, and may be adapted for practicing a variety of mathematical skills.

BIG NUMBER GAME

Start by telling the students how many digits there will be in the final number. Instruct them to draw a corresponding number of horizontal blanks on their paper, as if they were about to play a game of hangman. Tell them that you will be calling out digits from zero to nine, but you will never call the same digit twice in the same game. Students must try to place the digits in a way that will create the largest number. Call the digits out one at a time. After you call a digit, each student writes that digit in one of the spaces on his or her paper. Make sure that all the students record the digit in one of their available spaces before you call the next digit. Record the digits on the chalkboard as you call them out.

After all the spaces are filled, have students help you create the largest number possible from the available digits. Then have them estimate how close their number is to the largest possible number. Next, each student should find the actual difference between their number and the largest number, and compare that result to their estimate. The winner of the round is the student in the group who has the largest number and can correctly read it aloud. The winner gets to call out the next round of numbers.

There are several variations on this same idea. Decimal Big Number follows the same set of rules, but a decimal point is placed between two of the spaces. Fraction Big Number has spaces in the numerator and the denominator, and students must use division to convert the fraction into a decimal to determine a winner. Small Number is yet another option.

BASE (TEN) BALL

Place value may be practiced using a ball with the numerals 1 through 9 written on it. The ball is tossed to a student who catches it and reports the first number that he or she sees. The student then performs a predetermined operation, such as adding 10 to that number. Initially it may be helpful to demonstrate by holding a base-ten stick (or rod) to the left of the ball (in the tens place). This demonstrates the addition of 10 without the need to count or disturb the ones place. This exercise makes a great icebreaker activity and can be adapted to a large number of situations.

Handwriting Skills

Before addressing multidigit algorithms, it is important to deal with the prerequisite skills of numeral formation and spacing. Deficiencies in fine motor skills, visual-motor integration (VMI), and spatial perception skills may manifest themselves in poor numeral formations. Individual numerals might be illegible, or numerals within the same number may vary drastically in size. Disorganized placement on the page may also occur.

These issues have an impact even at the level of single-step arithmetic because students are unable to decode and rehearse their written text. In addition, teachers who are unable to decode written text may tend to consider it wrong, even if the child produced an accurate answer at the verbal level. These issues need to be addressed as soon as possible. Such students receive negative feedback for correct responses and risk developing math anxiety.